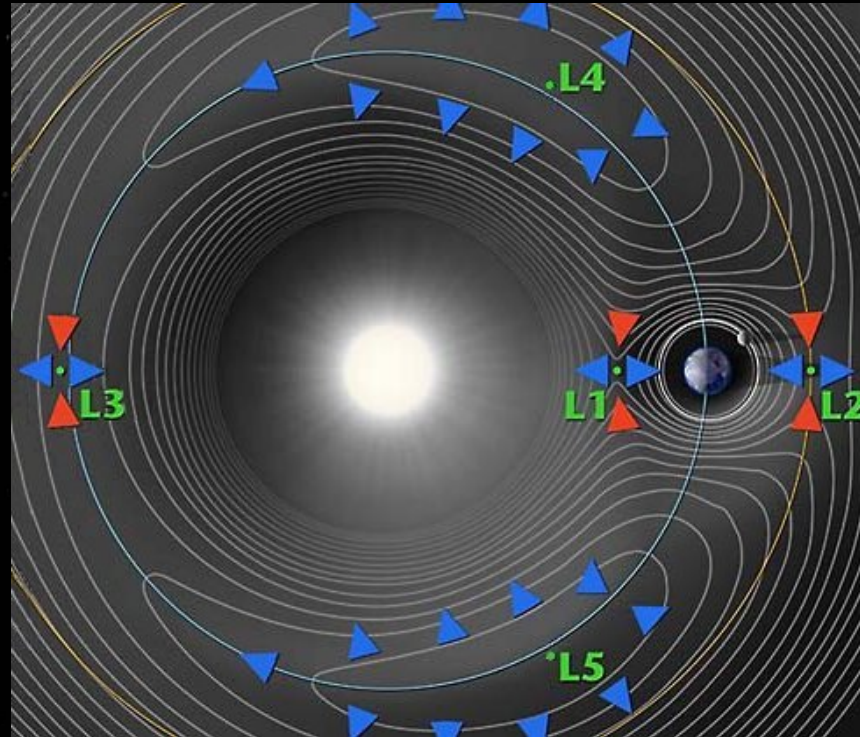


SOHO

(Solar and Heliospheric Observatory)



SOHO orbits the first Lagrange point

ESA - NASA collaboration



Last lecture (4)

- Solar wind
 - magnetic structure
- Ionosphere
 - ionospheric layers

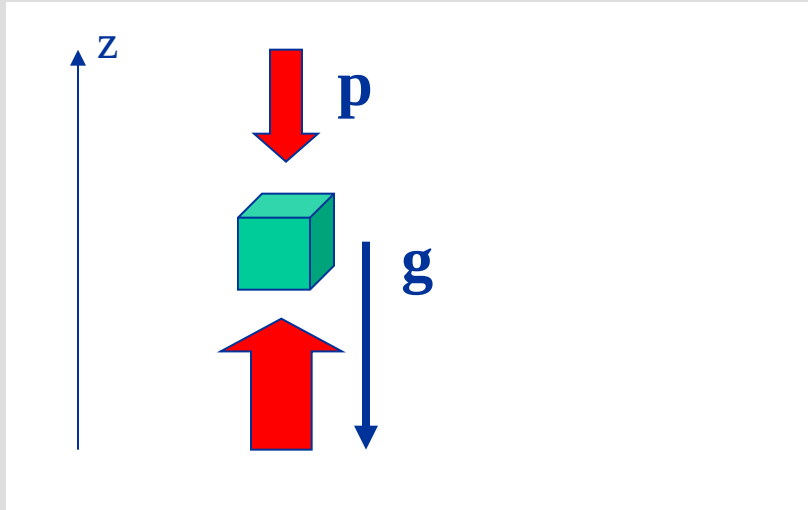
Today's lecture (5)

- Ionosphere
 - index of refraction
 - reflection of radio waves
 - particle drift motion in magnetized plasma
 - electrical conductivity in magnetized plasma
- Magnetosphere?



Today

Activity	Date	Time	Room	Subject	Litterature
L1	2/9	10-12	Q33	Course description, Introduction, The Sun 1, Plasma physics 1	CGF Ch 1, 5, (p 110-113)
L2	4/9	10-12	Q21	The Sun 2, Plasma physics 2	CGF Ch 5 (p 114-121), 6.3
L3	8/9	13-15	Q36	Solar wind, The ionosphere and atmosphere 1, Plasma physics 3	CGF Ch 6.1, 2.1-2.6, 3.1-3.2, 3.5, LL Ch III, Extra material
T1	10/9	10-12	Q33	Mini-group work 1	
L4	15/9	13-15	Q31	The ionosphere 2, Plasma physics 4	CGF Ch 3.4, 3.7, 3.8
T2	17/9	10-12	Q33	Mini-group work 2	
L5	19/9	15-17	Q31	The Earth's magnetosphere 1 , Plasma physics 5	CGF 4.1-4.3, LL Ch I, II, IV.A
L6	23/9	8-10	Q31	The Earth's magnetosphere 2, Other magnetospheres	CGF Ch 4.6-4.9, LL Ch V.
T3	24/9	14-16	Q21	Mini-group work 3	
L7	29/9	11-13	Q36	Aurora, Measurement methods in space plasmas and data analysis 1	CGF Ch 4.5, 10, LL Ch VI, Extra material
T4	1/10	15-17	Q31	Mini-group work 4	
L8	2/10	15-17	Q34	Space weather and geomagnetic storms	CGF Ch 4.4, LL Ch IV.B-C, VII.A-C
L9	8/10	13-15	Q36	Interstellar and intergalactic plasma, Cosmic radiation, Swedish and international space physics research.	CGF Ch 7-9
T5	9/10	15-17	Q31	Mini-group work 5	
L10	13/10	15-17	Q33	Guest lecture (preliminary): Swedish astronaut Christer Fuglesang	
T6	16/10	10-12	Q36	Round-up	
Written examination	30/10	8-13	M33, M37, M38		



Atmospheric scale height

$$-\frac{dp}{dz} = g\rho_m \quad \text{hydrostatic equilibrium for a volume element}$$

$$p = nk_B T = \frac{\rho k_B T}{m} \quad \text{ideal gas law}$$

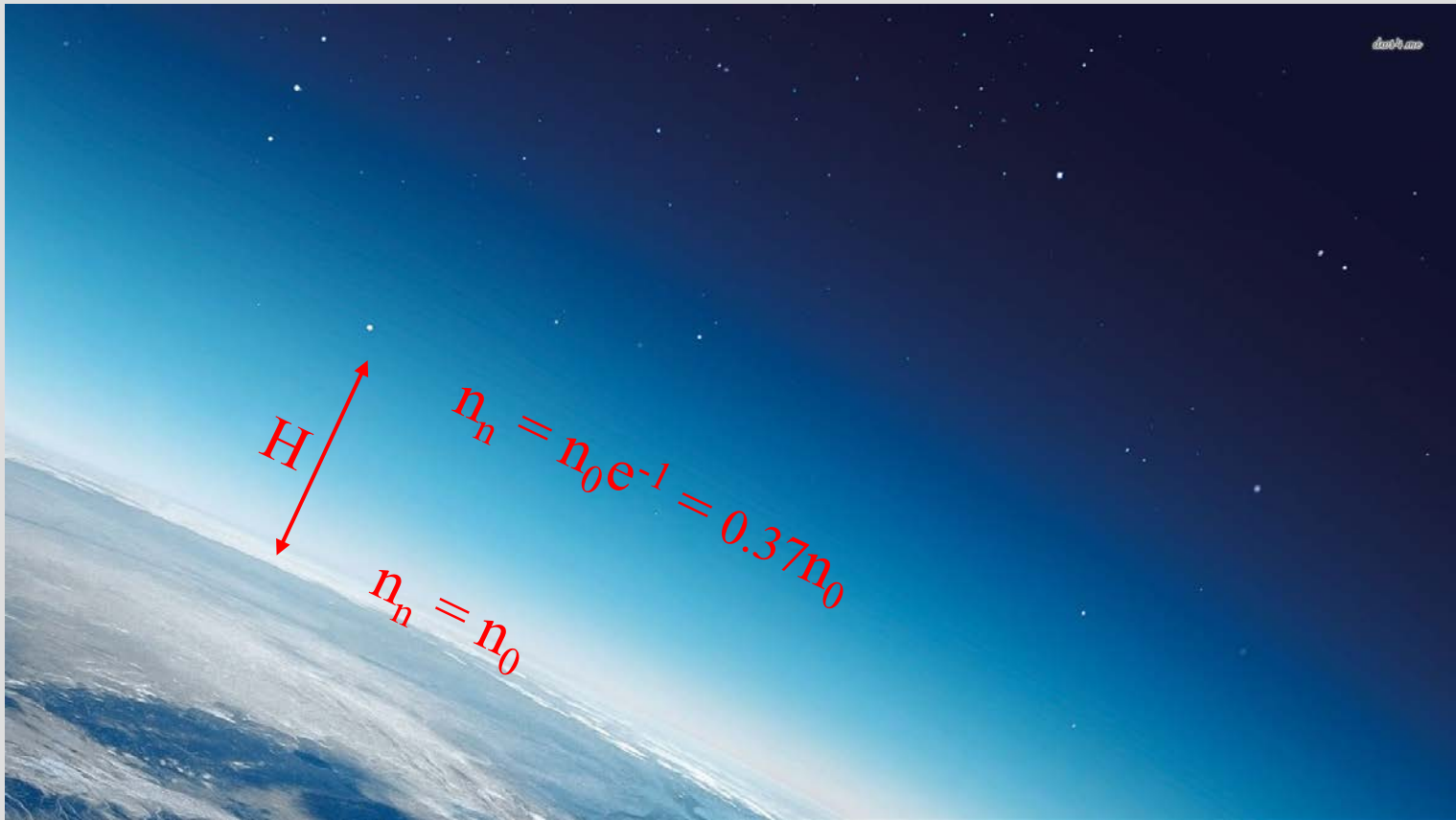
$$-\frac{k_B T}{m} \frac{d\rho_m}{dz} = g\rho_m \quad \text{if } T \text{ is constant}$$

$$\rho_m = \text{const} \cdot e^{-z/(k_B T / gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T / gm$$

Atmospheric scale height



$$\rho_m = \text{const} \cdot e^{-z/(k_B T / gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T / gm$$

Continuity equation

$$\frac{dn_e}{dt} = q - r$$

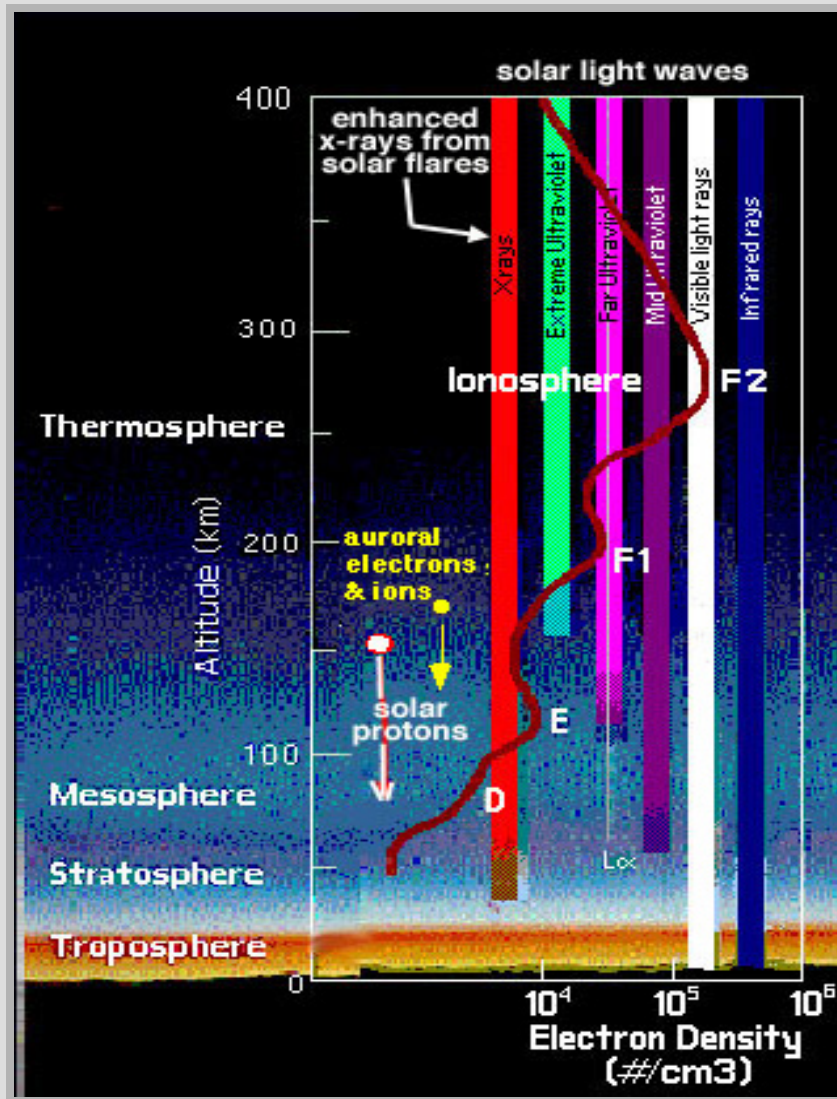
$$q = a_i I n_n$$

Ionization ($\text{m}^{-3}\text{s}^{-1}$)

Recombination ($\text{m}^{-3}\text{s}^{-1}$)

$$r = a_r n_e n_i = a_r n_e^2$$

Example: $e + \text{O}_2^+ \rightarrow \text{O} + \text{O}$ (dissociative recombination)



UV and X-ray radiation

$$\frac{dI}{dz} = In_n a_a$$

Electron density in Chapman layer

$$n_e = \left\{ \frac{a_i}{a_r} I_0 n_0 e^{-\left(H a_a n_0 e^{-z/H} + z/H \right)} \right\}^{1/2}$$



What does it look like in reality?

- Temperature not constant
- Many different wavelengths in solar radiation
- Several different molecules and atoms in neutral atmosphere. Composition also depends on altitude.

"E-region" - simple model calculation

O₂ dominating species, 10 nm X-ray radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

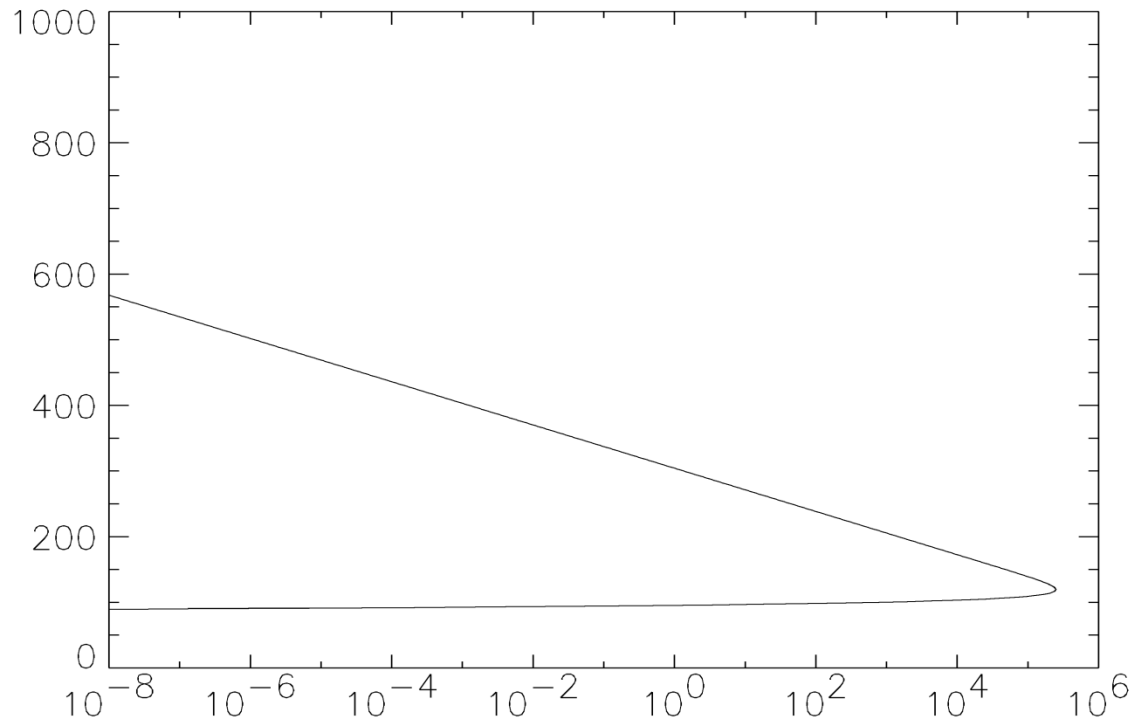
$$a_r = 3.0 \times 10^{-14}$$

$$T = 270$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 3.6 \times 10^{13} \text{ photons/m}^2/\text{s}$$



"F1-region" - simple model calculation

O₂ dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

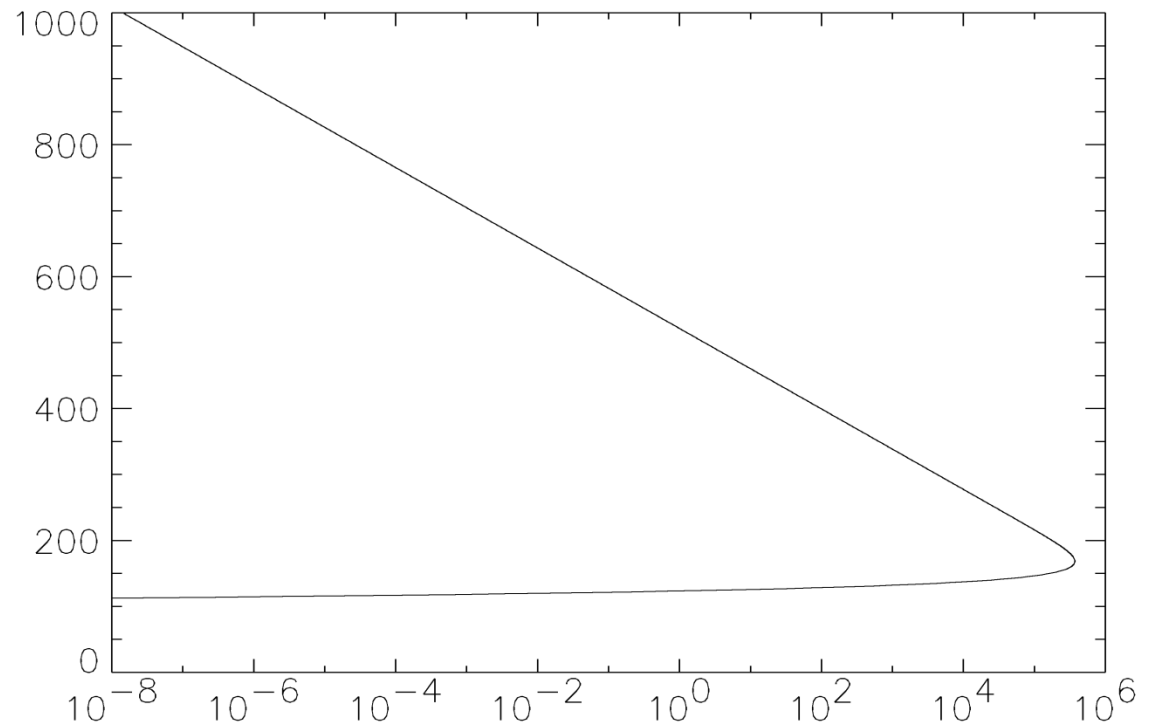
$$a_r = 3.0 \times 10^{-14}$$

$$T = 500$$

$$m = 16 \cdot 2 \cdot \text{amu}$$

$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



"F2-region" - simple model calculation

O dominating species, 30 nm UV radiation

$$a_a = 9.3 \times 10^{-23}$$

$$a_j = 9.3 \times 10^{-23}$$

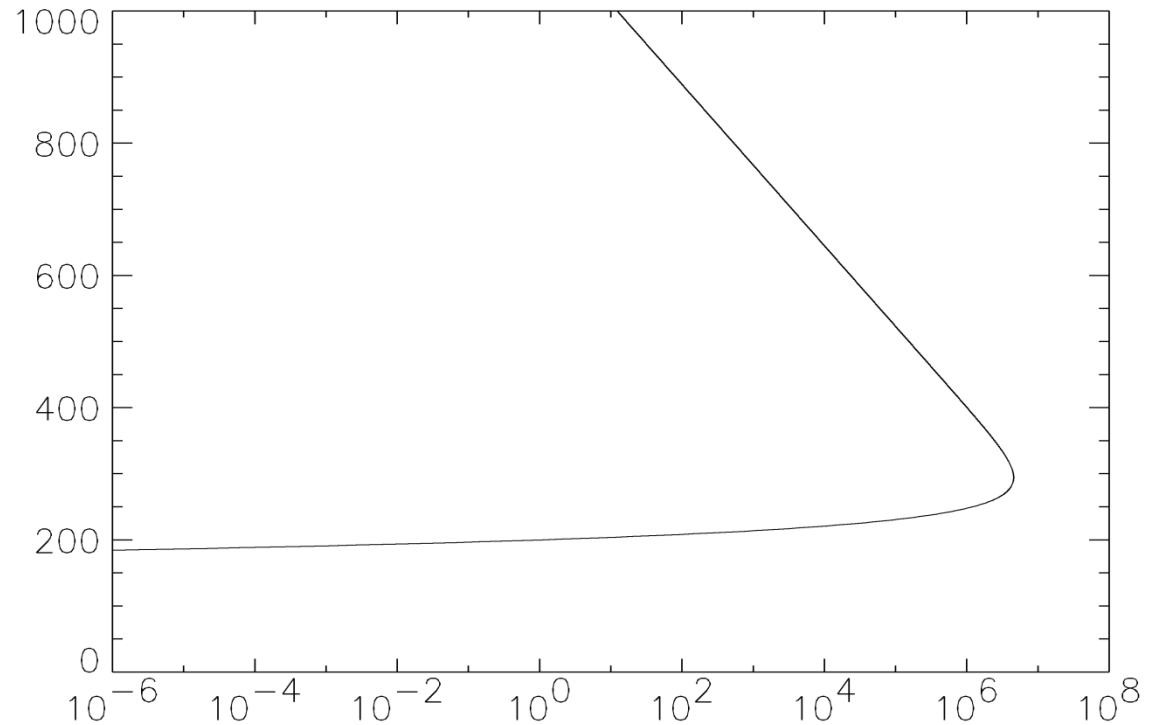
$$a_r = 1.0 \times 10^{-16}$$

$$T = 500$$

$$m = 16 \text{ amu}$$

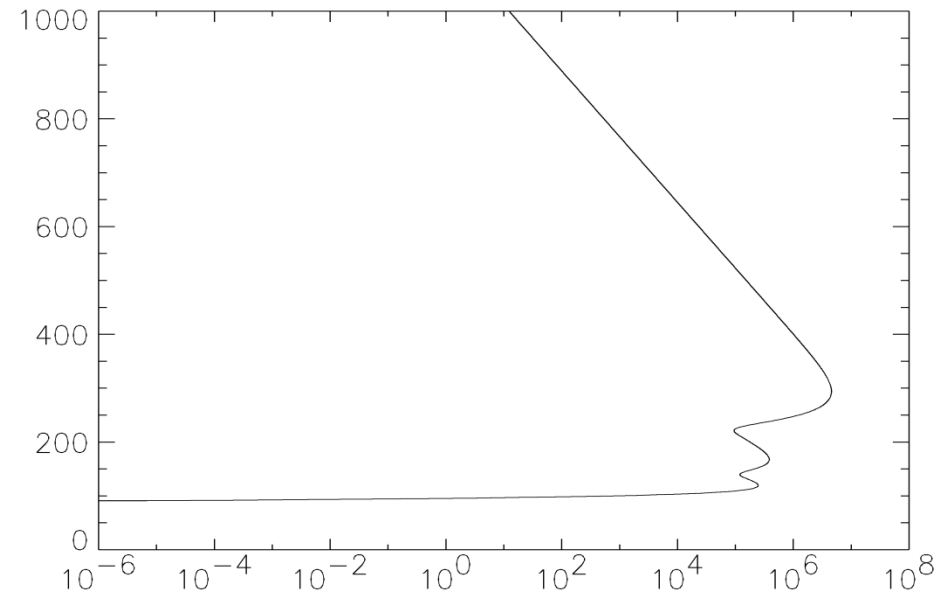
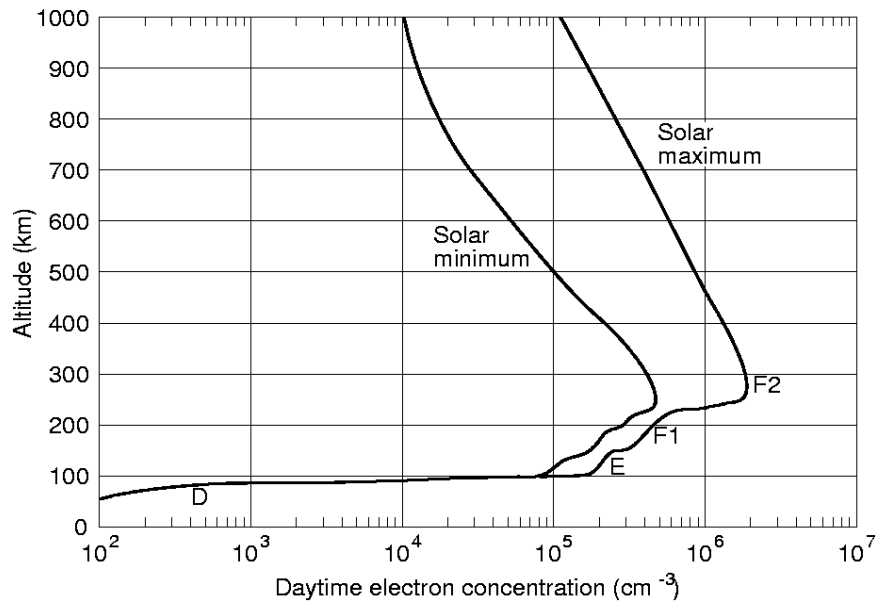
$$n_0 = 2.7 \times 10^{25} \text{ m}^{-3}$$

$$I_0 = 1.5 \times 10^{14} \text{ photons/m}^2/\text{s}$$



Measurements

"E" + "F1" + "F2"



Ionospheric layers

Layer	D	E	F ₁	F ₂
Altitude (km)	60-85	85-140	140-200	200 - ca 1500
Nighttime electron density (cm ⁻³)	<10 ²	2 · 10 ³	—	2 - 5 · 10 ⁵
Daytime electron density (cm ⁻³)	10 ³	1 - 2 · 10 ⁵	2 - 5 · 10 ⁵	0.5 - 2 · 10 ⁶
Ion species	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺	NO ⁺ O ₂ ⁺ O ⁺	O ⁺ He ⁺ H ⁺
Cause of ionization	Lyman α (1215 Å) + cosmic radiation	Lyman β (1025 Å) X-rays	UV	UV

NO⁺ created by chemical reaction $\text{N}_2^+ + \text{O} \rightarrow \text{NO}^+ + \text{N}$



Propagation of radio waves in the ionosphere

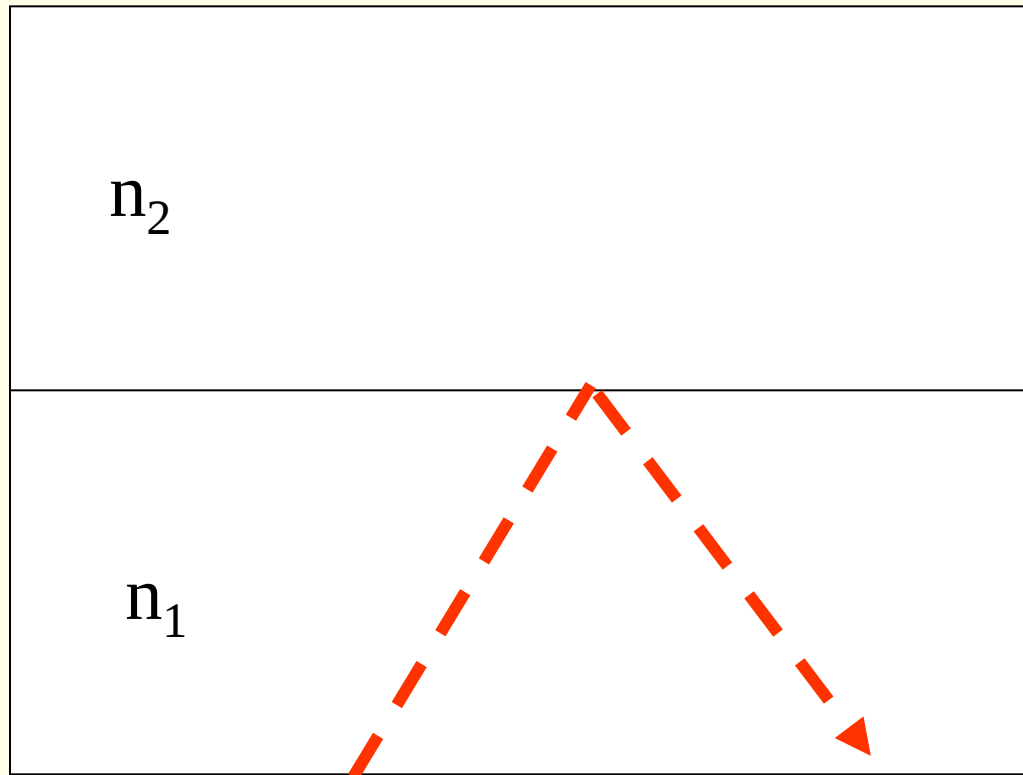
1. Absorption/damping

Takes place in the D-region due to high collision frequency. (Collisions with neutral atoms.)

2. Reflection

takes place in the F-region due to large gradients in the refraction index.

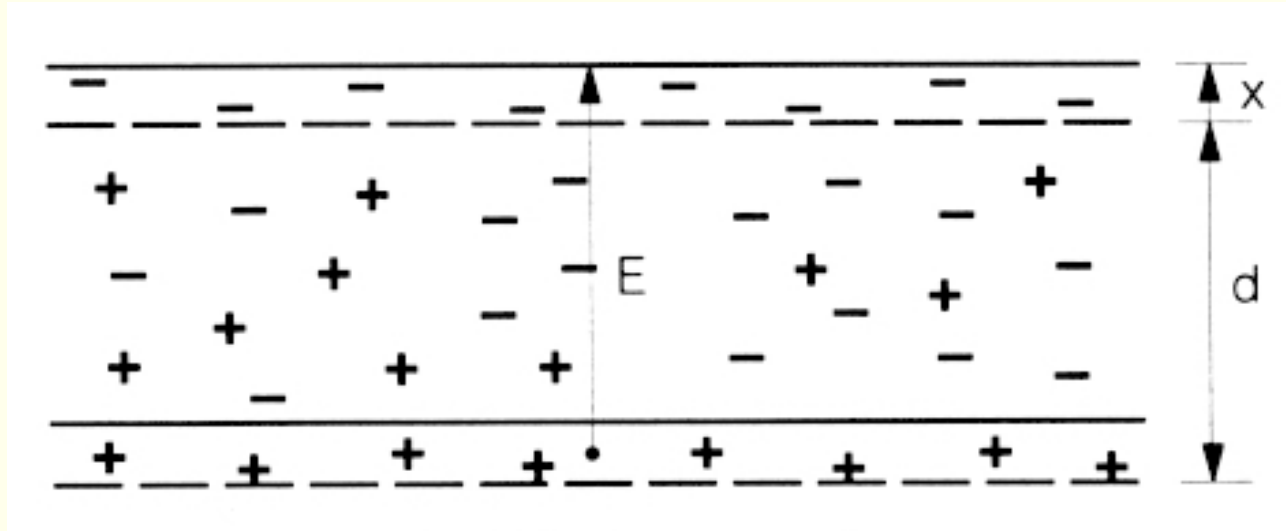
Reflection of radio waves



Total reflection at a sharp boundary (or large gradient) if

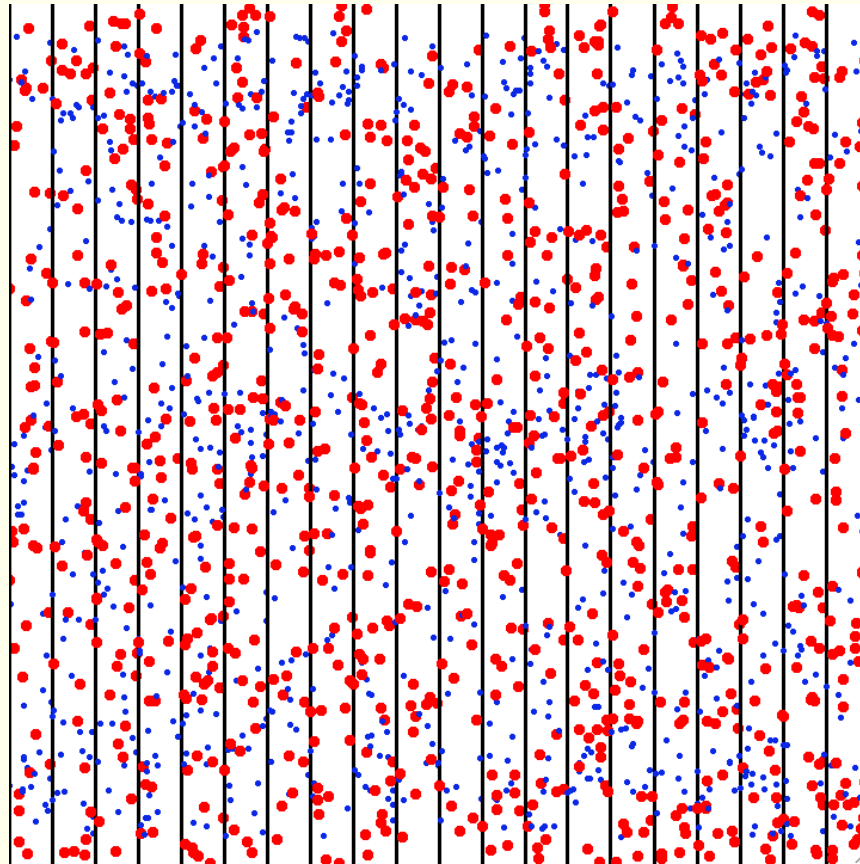
$$n_2 < n_1$$

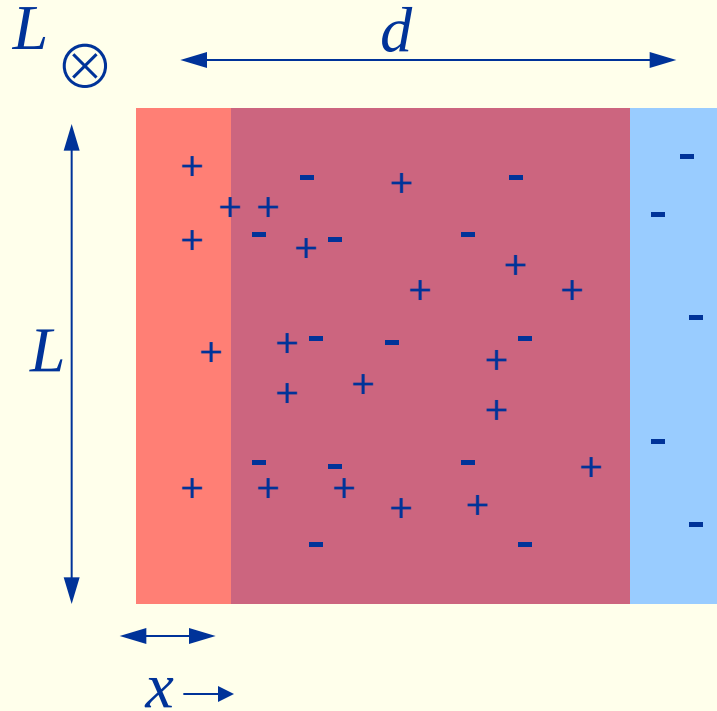
Plasma frequency



Charge imbalance creates an electric field which tends to even out the imbalance.

Plasma oscillations parallel to B





Newtons law on an individual electron inside the slab:

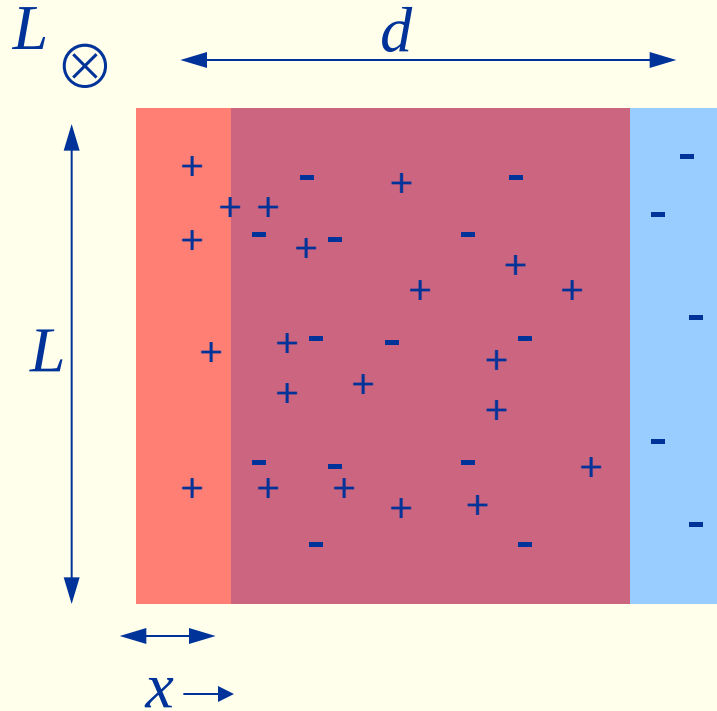
$$F = m_e a$$

$$F = -eE$$

$$E = \frac{\sigma}{\epsilon_0}$$

Surface charge density

$$\sigma = -en_e x$$

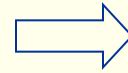


$$F = m_e a$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$F = -eE$$

$$\sigma = en_e x$$



$$-\frac{n_e e^2 x}{\epsilon_0 m_e} = \frac{d^2 x}{dt^2}$$

$$x = \sin(\omega_{pe} t)$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

What is the plasma frequency f_{pe} at the daytime E-region, close to solar minimum? (see Fälthammar p 28)

$$f_{pe} = \frac{\omega_{pe}}{2\pi} \equiv \frac{1}{2\pi} \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

Blue

7 kHz

Yellow

400 MHz

Green

3 MHz

Red

2 GHz

Index of refraction for electromagnetic waves in a plasma

$$(1) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(3) \quad \mathbf{j} = -en_e \mathbf{v}_e$$

$$(4) \quad m_e \frac{\partial \mathbf{v}_e}{\partial t} = -e\mathbf{E}$$

Assume all quantities vary sinusoidally, with frequency ω , e.g.:

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$(1) \Rightarrow \nabla \times (\nabla \times \mathbf{E}) = -\nabla \times \frac{\partial \mathbf{B}}{\partial t}$$

$$(2) \Rightarrow \nabla \times \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \frac{\partial \mathbf{j}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\therefore \nabla \times (\nabla \times \mathbf{E}) = -\mu_0 \frac{\partial}{\partial t} (en_e \mathbf{v}_e) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

$$\nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 en_e \frac{\partial \mathbf{v}_e}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$\Rightarrow$

Index of refraction for electromagnetic waves in a plasma

$$\cancel{ik(ik \cdot \mathbf{E})} - k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \mathbf{v}_e + \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$

Does not represent E.M. wave

(4) $\Rightarrow$

$$-k^2 \mathbf{E} = \mu_0 (-i\omega) en_e \frac{ie\mathbf{E}}{\omega m_e} + \frac{1}{c^2} (-i\omega)^2 \mathbf{E}$$

$\Rightarrow$

$$c^2 k^2 = -c^2 \frac{\mu_0 n_e e^2}{m_e} + \omega^2 = \frac{-1}{\cancel{\mu_0 \epsilon_0}} \frac{\cancel{\mu_0} n_e e^2}{m_e} + \omega^2$$

$$\therefore \omega^2 = c^2 k^2 + \omega_p^2$$

$$n^2 = \frac{c^2}{v_{ph}^2} = \frac{c^2 k^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}$$

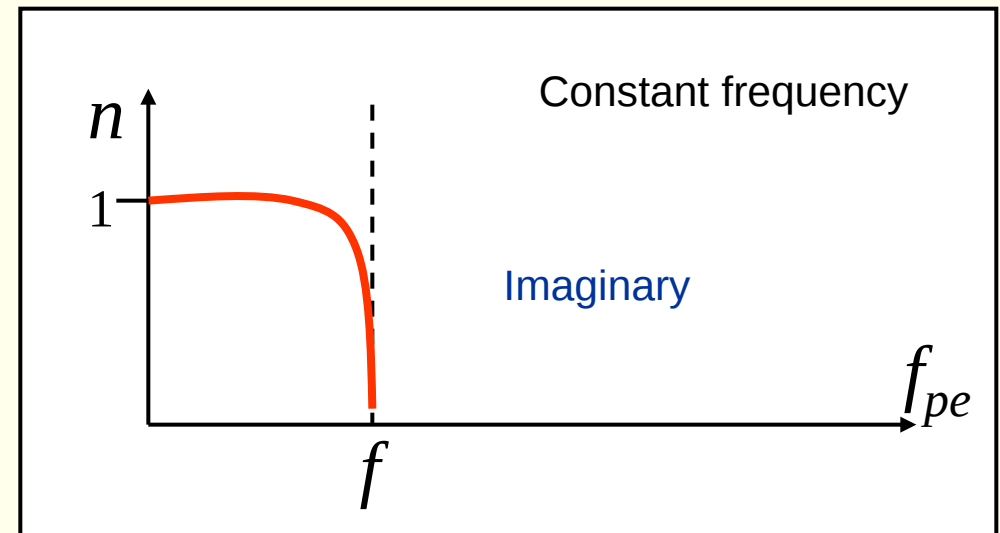
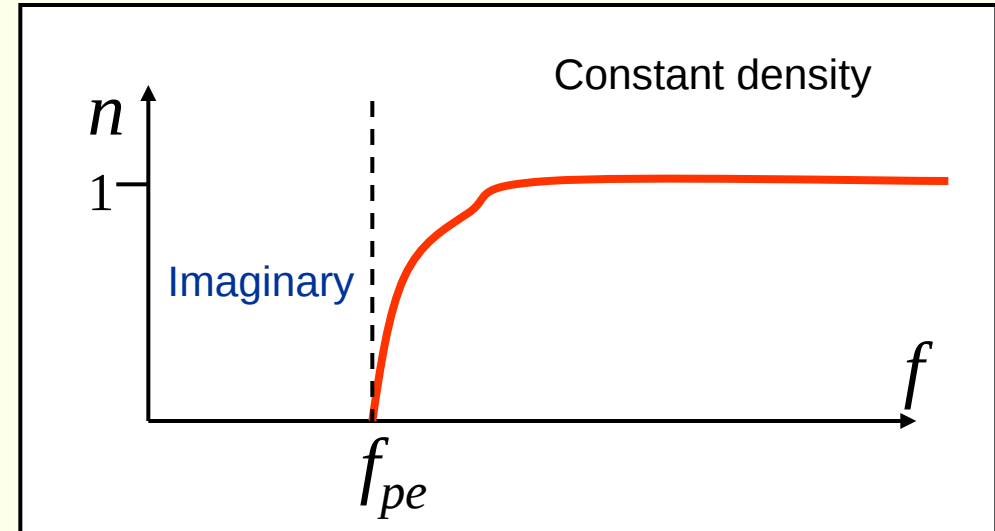
$\therefore$

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = \sqrt{1 - \frac{f_p^2}{f^2}}$$

Refraction index for plasma

$$n = \frac{c}{v_{ph}} = \sqrt{1 - \frac{f_{pe}^2}{f^2}}$$

$$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$$

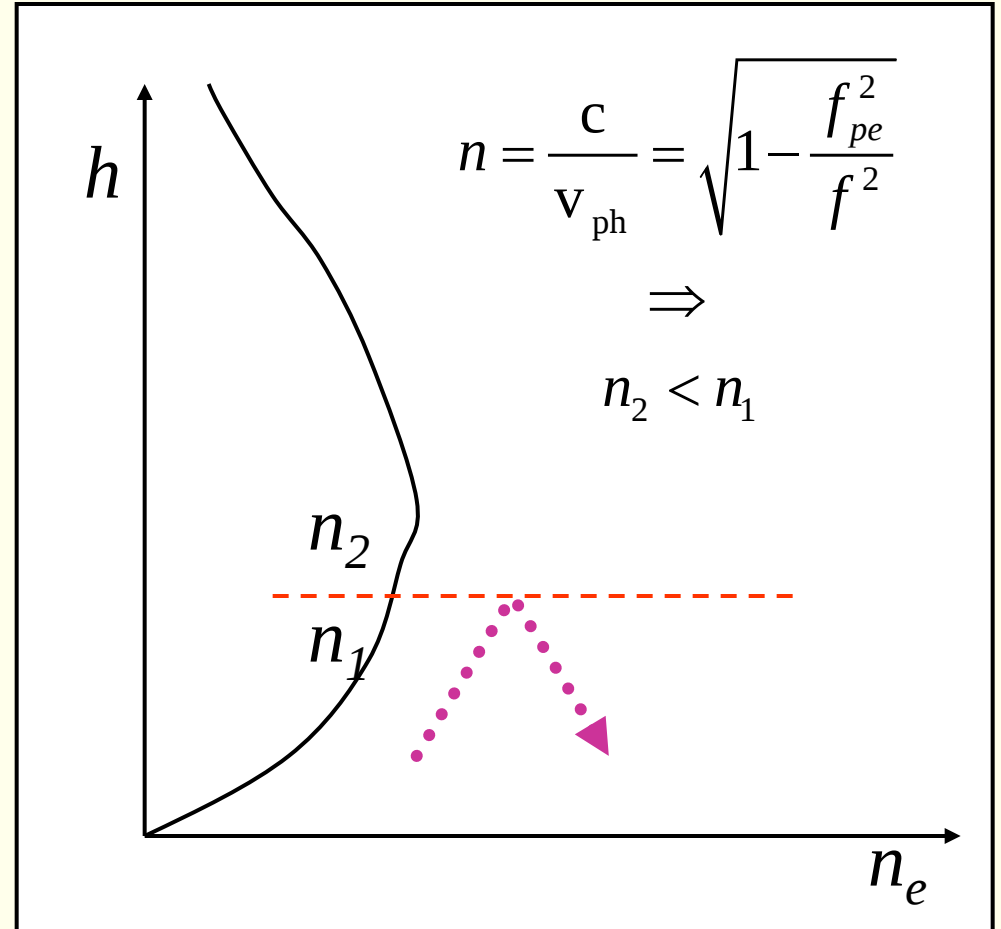


Where does the total reflection take place?

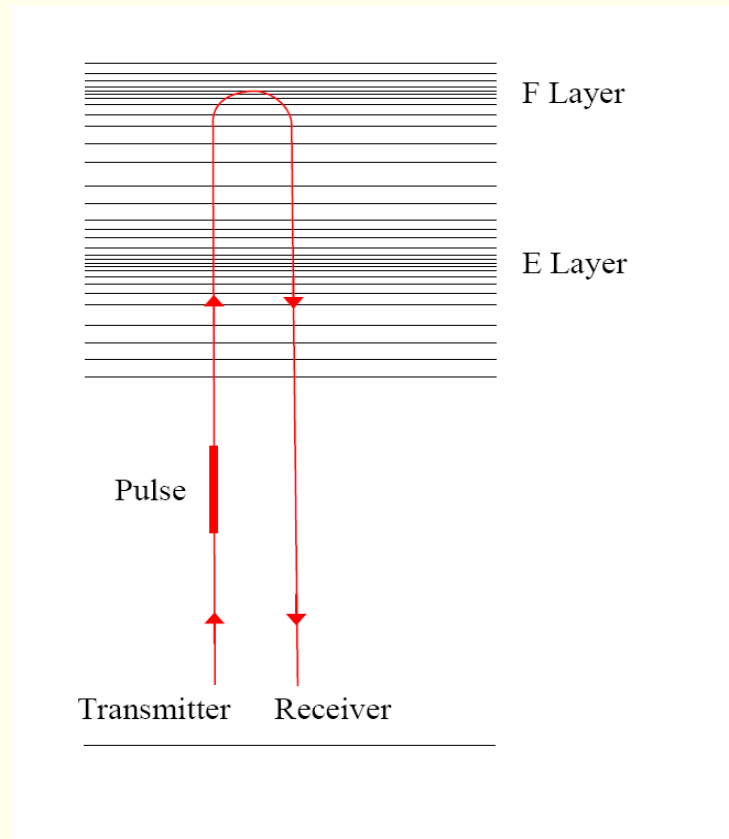
Large gradient when

$$f \approx f_{pe}$$

Higher frequencies $\rightarrow$ higher $f_{pe}(n_e)$



Ionosonde



The pulse will be reflected where

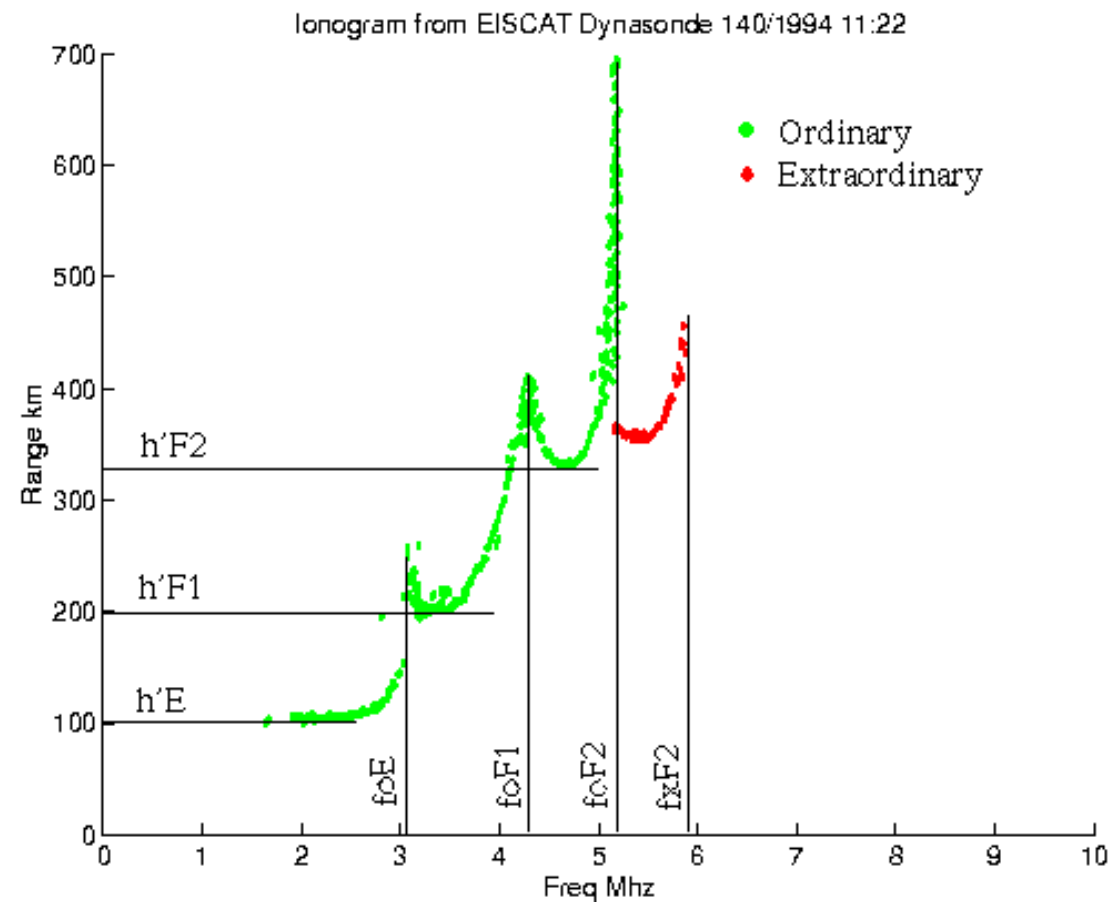
$$f = f_{pe}$$

The altitude will be determined by

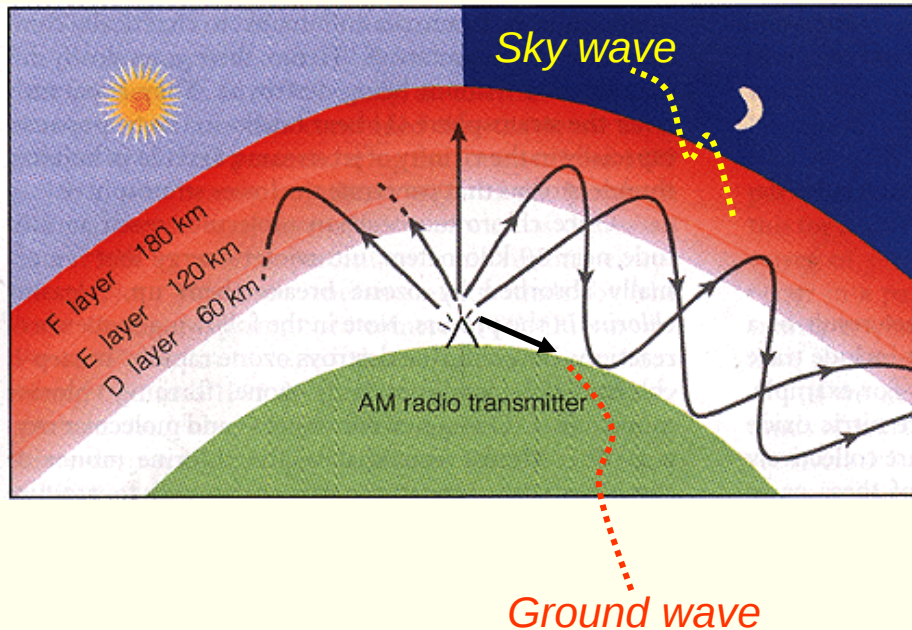
$$2h = ct$$

Where t is the time between when the pulse is sent out and the registered again.

Ionogram



Reflection of radio waves



F2-layer during night:

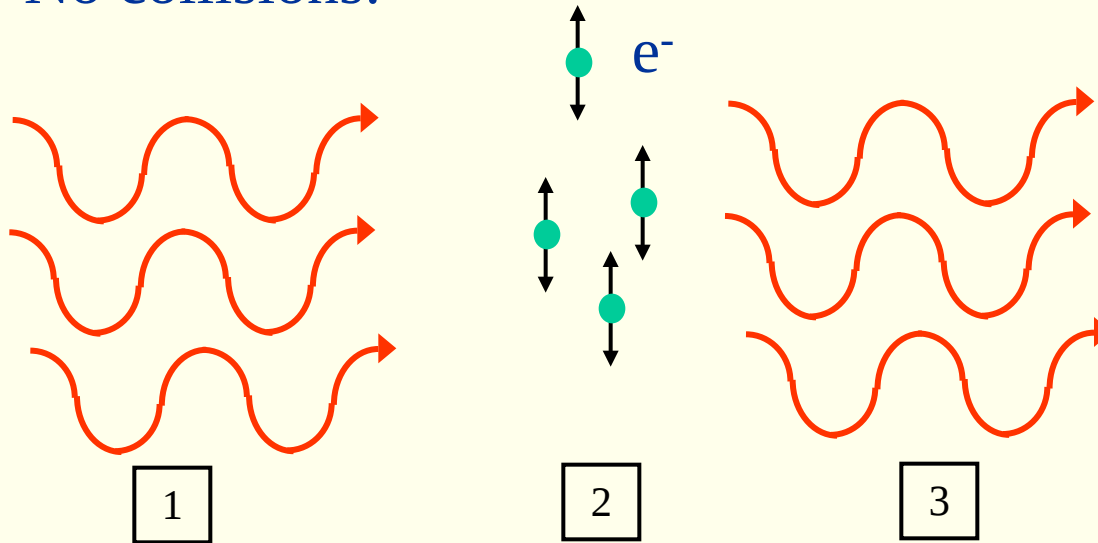
$$n_e = 5 \cdot 10^{11} \text{ m}^{-3} \Rightarrow$$

$$f_{pe} = 10^7 \text{ Hz} = 10 \text{ MHz}$$

= HF/short wave

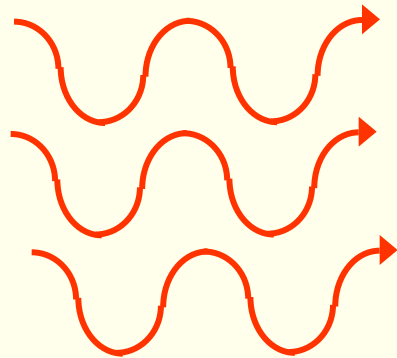
Absorption of radio waves

No collisions:

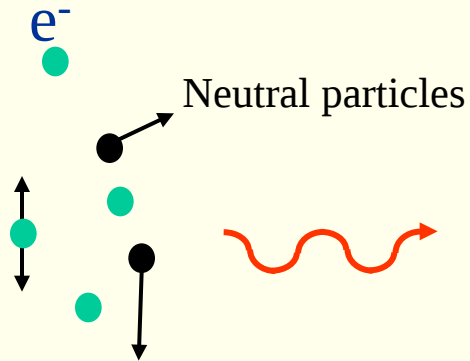


Absorption of radio waves

With collisions:



1



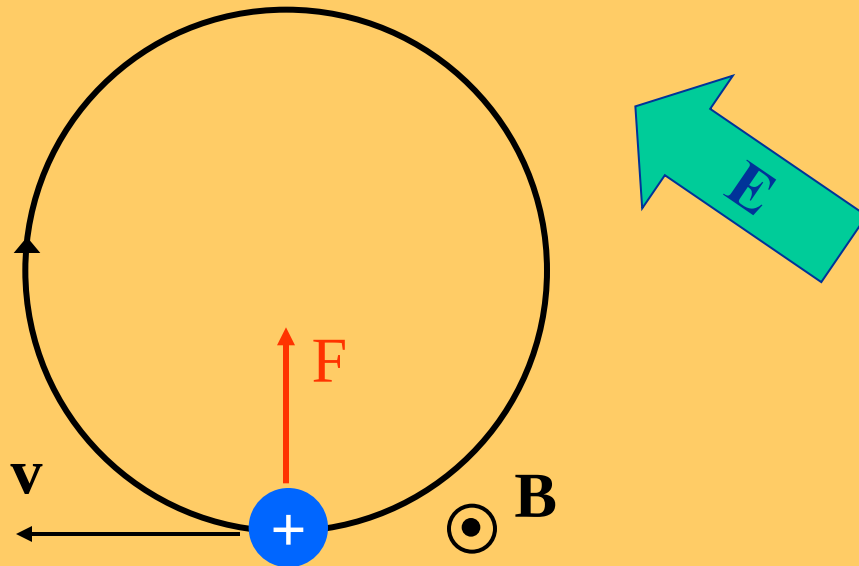
2

3

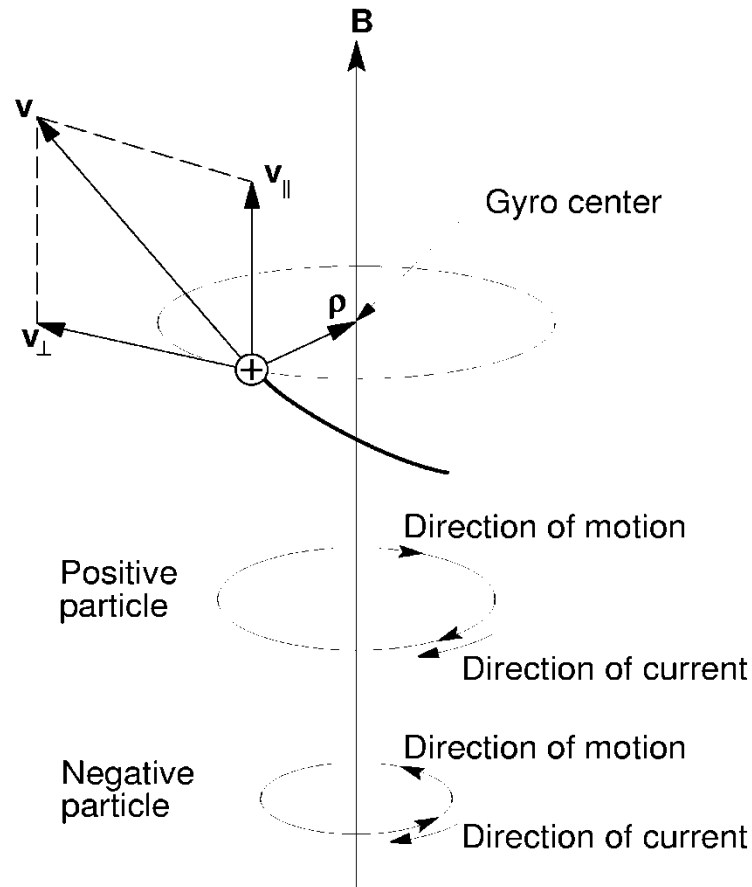
Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

What happens if you add an electric field $\mathbf{E}$?



Particle motion in magnetic field



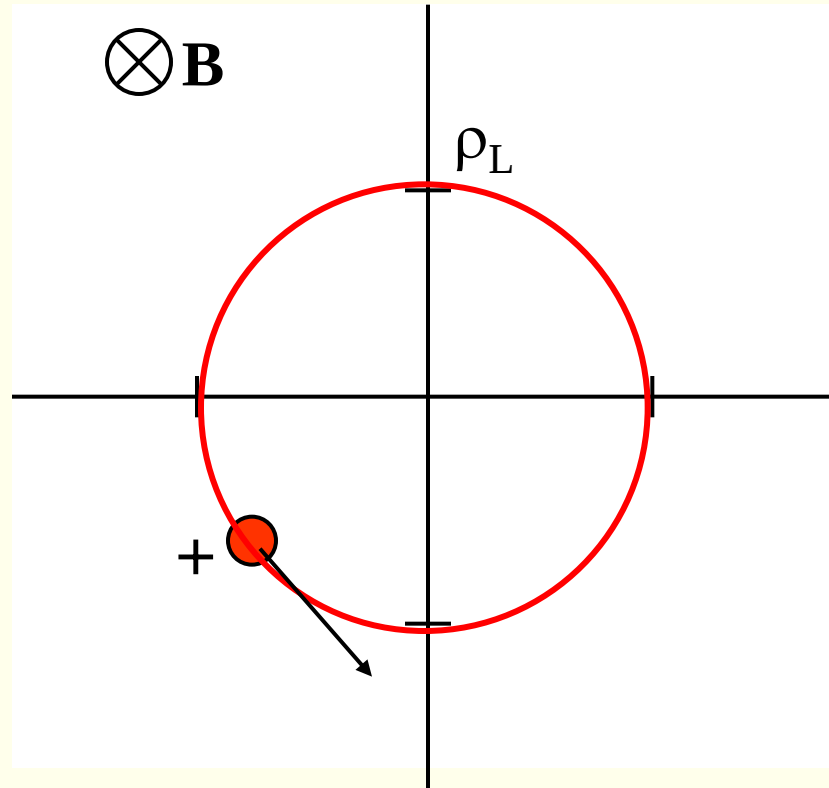
gyro radius

$$\rho = \frac{mv_{\perp}}{qB}$$

gyro frequency

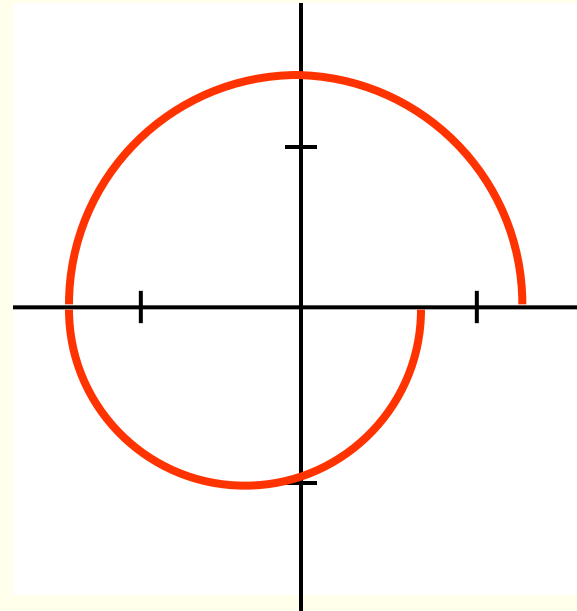
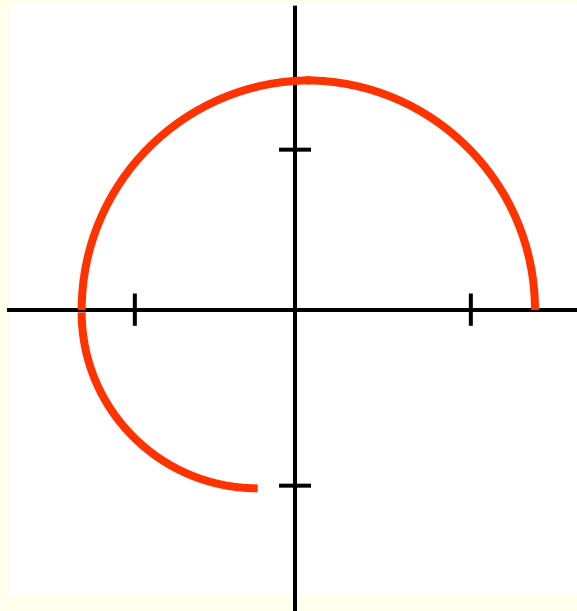
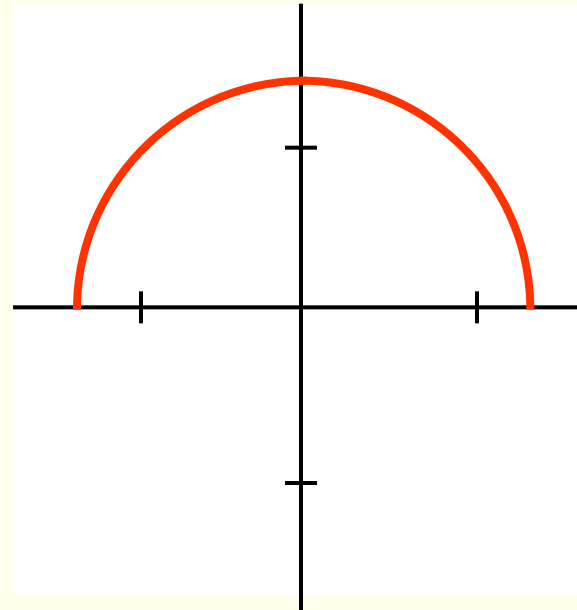
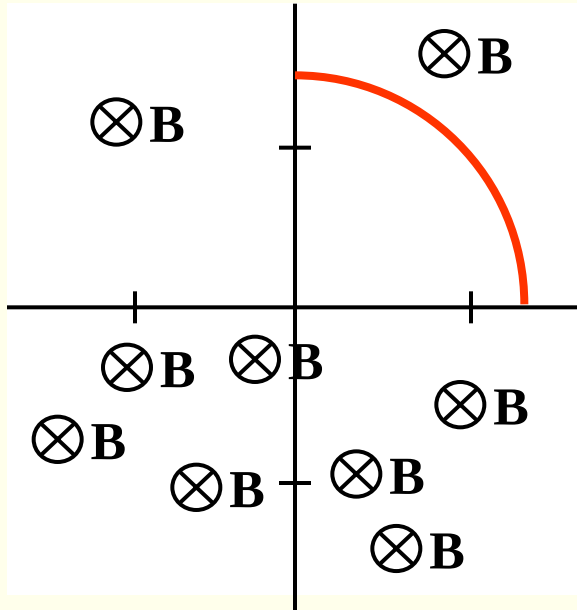
$$\omega_g = \frac{qB}{m}$$

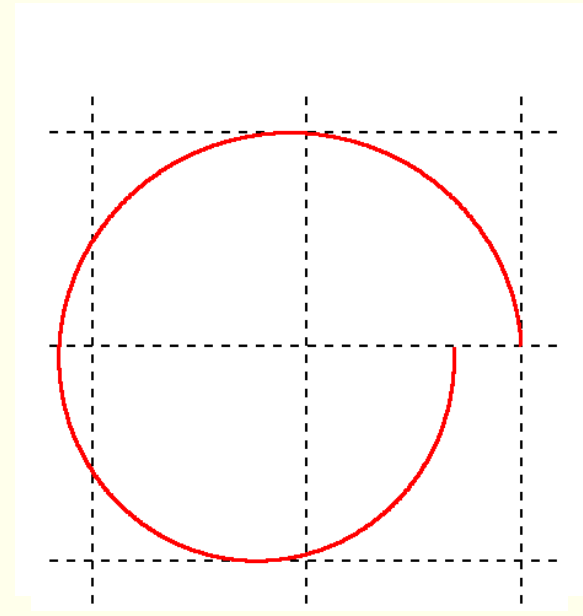
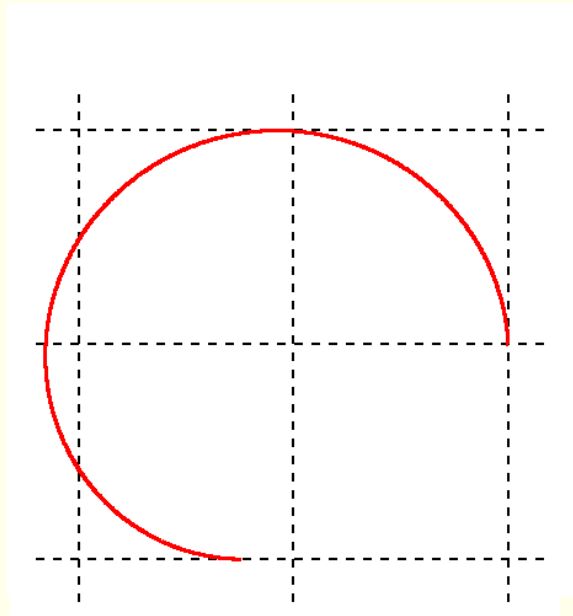
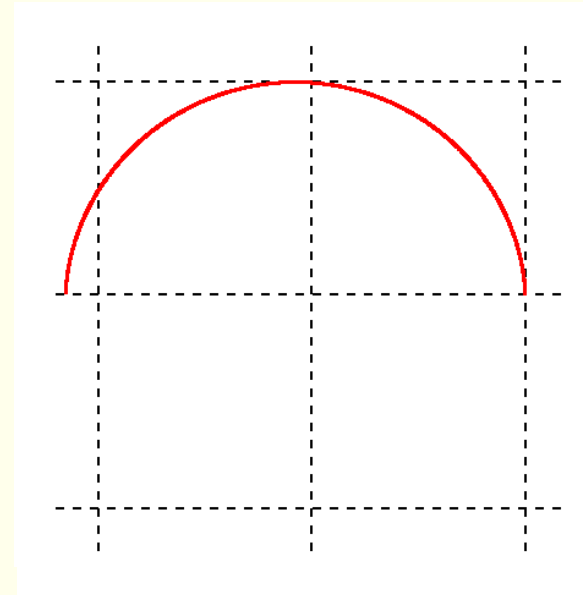
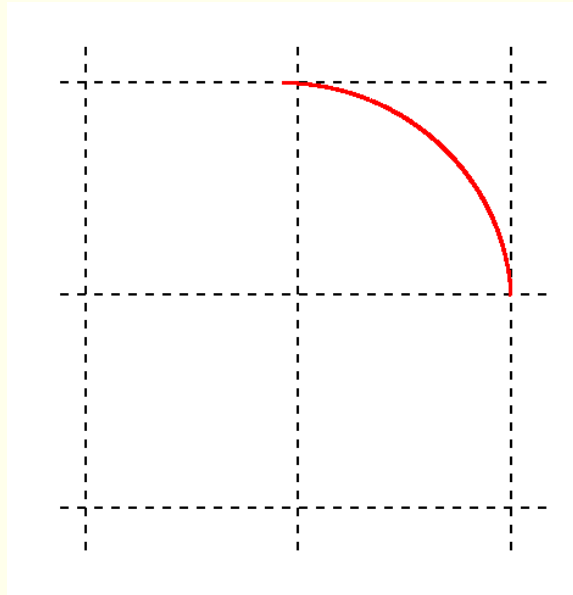
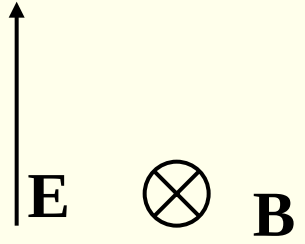
Drift motion





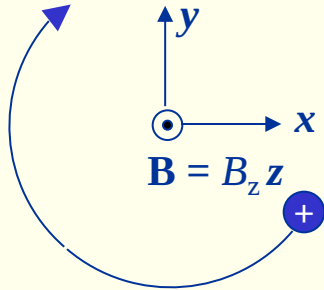
$$\rho = \frac{mv_{\perp}}{qB}$$





Drift motion

Consider a charged particle in a magnetic field.



Assume an electric field in the x-z plane:

$$\mathbf{E} = (E_x, 0, E_z)$$

$$m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B} + \mathbf{E}) \Rightarrow$$

$$\begin{cases} m \frac{dv_x}{dt} = qv_y B + qE_x \\ m \frac{dv_y}{dt} = -qv_x B \\ m \frac{dv_z}{dt} = qE_z \end{cases} \quad \text{Constant acceleration along } z$$



$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$



Drift motion

$$\begin{cases} \frac{d^2 v_x}{dt^2} = \frac{qB}{m} \frac{dv_y}{dt} = \omega_g \frac{dv_y}{dt} = -\omega_g^2 v_x \\ \frac{d^2 v_y}{dt^2} = -\frac{qB}{m} \frac{dv_x}{dt} = -\omega_g \frac{dv_x}{dt} = -\omega_g^2 v_y - \frac{q^2 B}{m^2} E_x \end{cases}$$

$\therefore$

$$\begin{cases} \frac{d^2 v_x}{dt^2} - \omega_g^2 v_x \\ \frac{d^2 \left(v_y + \frac{E_x}{B} \right)}{dt^2} = -\omega_g^2 \left(v_y + \frac{E_x}{B} \right) \end{cases}$$



$$\begin{cases} v_x = v_{\perp} e^{i\omega_g t + \delta_x} \\ v_y = -\frac{E_x}{B} + v_{\perp} e^{i\omega_g t + \delta_y} \end{cases}$$

Average over a gyro period:

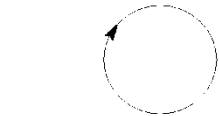
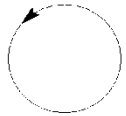
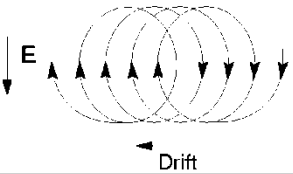
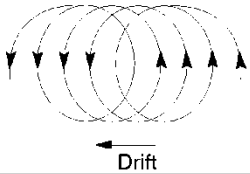
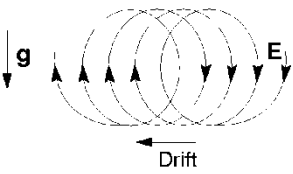
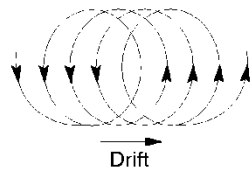
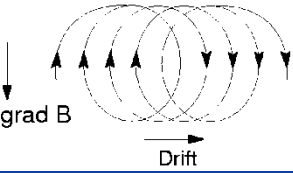
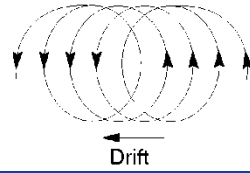
$$v_{drift,y} = -\frac{E_x}{B} = -\frac{E_x B_z}{B^2} = \frac{(\mathbf{E} \times \mathbf{B})_y}{B^2}$$

In general:

$$\mathbf{v}_{drift} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} = \frac{q\mathbf{E} \times \mathbf{B}}{qB^2} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Drift motion

$$\mathbf{u}_{drift} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

	Positive particles	Negative particles
Homogeneous magnetic field No disturbing force $\mathbf{F} = 0$		
Homogeneous magnetic field Homogeneous electric field $\mathbf{F} = q\mathbf{E}$		
Homogeneous magnetic field Gravitation $\mathbf{F} = m\mathbf{g}$		
Inhomogeneous magnetic field $\mathbf{F} = -\mu \text{ grad } B$		



Last Minute!